Certification of Matrix Interpretations in Coq

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WST

Outline

1. CoLoR
2. Formalization of matrix interpretations
   - Introduction to matrix interpretations
   - Monotone algebras
   - Matrices
   - Matrix interpretations
3. Certified competition

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CoLoR overview

CoLoR

CoLoR: Coq Library on Rewriting and Termination.
Goal: certification of termination proofs produced by various termination provers.

How to do that? CoLoR approach:

- TPG: common format for termination proofs.
- Tools output proofs in TPG format.
- CoLoR: a Coq library of results on termination.
- Rainbow: a tool for translation from proofs in TPG format to Coq proofs, using results from CoLoR.

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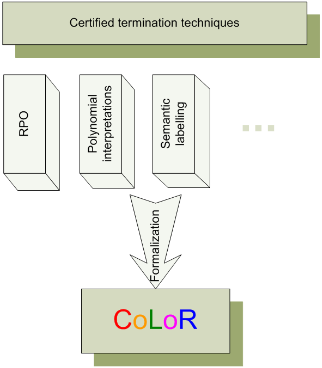

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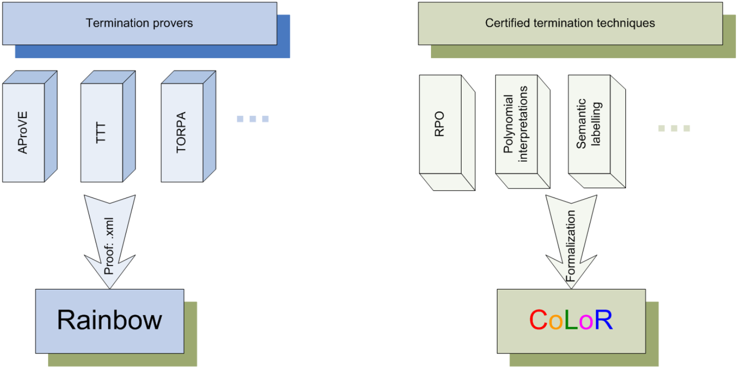

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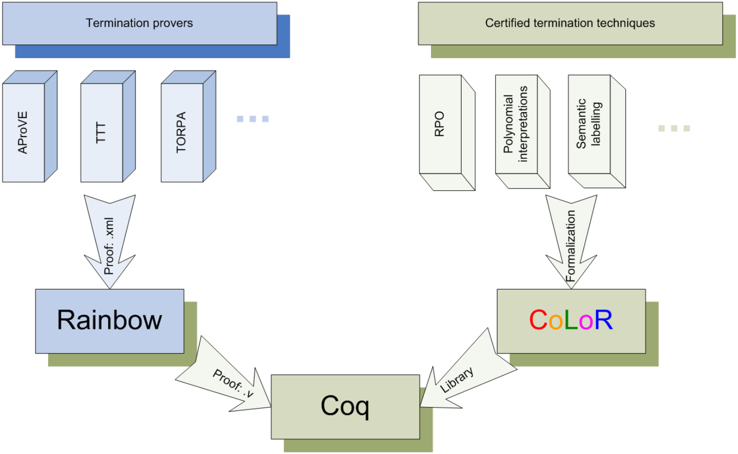

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Example

z086.trs

$a(a(x)) \to c(b(x)), \quad b(b(x)) \to c(a(x)), \quad c(c(x)) \to b(a(x))$

Matrix interpretation for z086.trs

$$a(x) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$$b(x) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

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Example ctd.

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Monotone algebras

Definition (An extended weakly monotone Σ-algebra)

A *weakly monotone Σ-algebra* $(A, [\cdot], >, \gtrsim)$ is a Σ-algebra $(A, [\cdot])$ equipped with two binary relations $>$, $\gtrsim$ on A such that:

- $>$ is well-founded;
- $> \cdot \gtrsim \subseteq >$;
- for every $f \in \Sigma$ the operation $[f]$ is monotone with respect to $>$.

Theorem

Let $\mathcal{R}, \mathcal{R}'$ be TRSs over a signature Σ, $(A, [\cdot], >, \gtrsim)$ be an extended monotone Σ-algebra such that:

- *$[\ell, \alpha] \gtrsim [r, \alpha]$ for every rule $\ell \to r$ in $\mathcal{R}$, for all $\alpha : \mathcal{X} \to A$ and*
- *$[\ell, \alpha] > [r, \alpha]$ for every rule $\ell \to r$ in $\mathcal{R}'$ and for all $\alpha : \mathcal{X} \to A$.*

Then $\mathrm{SN}(\mathcal{R})$ *implies* $\mathrm{SN}(\mathcal{R} \cup \mathcal{R}')$.

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Formalization of monotone algebras

- Monotone algebras are formalized as a functor.
- Apart for the aforementioned requirements there is one additional required to deal with concrete examples: $>_{\mathcal{T}}$ and $\gtrsim_{\mathcal{T}}$ must be decidable.
- More precisely the requirement is to provide a relation $\gg$, such that
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  - similarly for $\gtrsim$.
- The structure returned by the functor contains all the machinery required to prove (relative)-(top)-termination in Coq.

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Formalization of matrices

- Matrices over arbitrary semi-ring of coefficients.
- a number of basic operations over matrices such as:

$$[\cdot], \quad M_{i,j}, \quad M + N, \quad M * N, \quad M^T, \ \ldots$$

- and a number of basic properties such as:
  - $M + N = N + M$,
  - $M * (N * P) = (M * N) * P$
  - monotonicity of $*$
  - ...

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Polynomial interpretations in the setting of monotone algebras

- $A = \mathbb{Z}$,
- $> = >_{\mathbb{Z}}$, $\gtrsim = \geq_{\mathbb{Z}}$,
- interpretations represented by polynomials $[f(x_1, \ldots, x_n)] = P_{\mathbb{Z}}(x_1, \ldots, x_n)$,
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  where $M_i \in \mathbb{N}^{d \times d}$, $v \in \mathbb{N}^d$,
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- Participants:
  - CiME+ A3PAT (polynomial interpretations, LPO, DP)
  - TPA+ CoLoR (polynomial and matrix interpretations, DP)
  - $\mathsf{T_TT_2}$ + CoLoR (matrix interpretations, DP)
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The end

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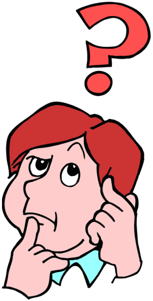

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